THERMAL BUCKLING BEHAVIOR OF THICK COMPOSITE P-FGM PLATES UNDER LINEAR TEMPERATURE RISE

M. BOUAZZA[1,2], A. TOUNSI [2], E.A. ADDA-BEDIA [2], A. MEGUENI [3]

[1] Department of Civil Engineering, University of Béchar, Béchar 08000, Algeria.
bouazza_mokhtar@yahoo.fr

[2] Laboratory of Materials and Hydrology (LMH), University of Sidi Bel Abbès, Sidi Bel Abbès 2200, Algeria.

[3] Department of Mechanical Engineering, University of Sidi Bel Abbès, Sidi Bel Abbès 2200, Algeria.

ABSTRACT

An analytical solution is presented for the buckling of the functionally graded plates are simply supported and subject to linear temperature rise. Based on the principles of Von Karman's theory and first order shear deformation theory , a set of governing equations for thermal buckling of thick rectangular plates together with the associated boundary conditions are derived Navier double series method is then applied to find the closed-form solutions. Effects of important parameters such as of ratio of length to width, ratio of length to thickness, etc., are studied.

KEY-WORDS : Thermal Buckling, Functionally Graded Material, First Order Shear Deformation Theory.

1. Introduction

The buckling of plates has been studied by many researchers. Solutions to isotropic plate buckling problems can be found in the monograph ofTimoshenko [1]. Buckling of composite plates has been investigated extensively in the monograph edited by Turvey and Marshall [2]. Following the use of laminated composite plates in engineering applications, bifurcation buckling of such structures has been investigated by many researchers without considering flatness before buckling [3]. This point was first clarified for laminated composite plates for some boundary conditions and for some lamina configurations by Leissa [3]. Qatu and Leissa [4] applied this result to identify true buckling behavior of composite plates.

Recent studies on new performance materials have led to a new material known as Functionally Graded Material (FGM). These are high-performance heat-resistant materials able to withstand ultrahigh temperatures and extremely large gradients used in spacecrafts and nuclear plants. FGMs are microscopically inhomogeneous where the mechanical properties vary smoothly and continuously from one surface to the other. These novel materials were first introduced in 1984 [5] and then developed by other scientists [6,7]. Typically, these materials are made from a mixture of ceramics and metal. It is apparent from the literature survey that most of the researches on FGMs have been restricted to thermal stress analysis, fracture mechanics, and optimization. Very little work has been done to consider the stability analysis, buckling, and vibrational behavior of FGM structures. Some research works related to the present study are introduced in the following.

Birman [8] studied the buckling problem of functionally graded composite rectangular plates subjected to uniaxial compression. Two classes of fibers are used in hybrid composite material. Linear equations of equilibrium for a symmetrically laminated plate which are uncoupled, have been derived and then solved to obtain the critical buckling load for simply supported edges condition.

Bouazza et al [9] studied buckling of functionally graded plates subjected to uniform temperature rise. The material properties of the FGM plates are assumed to change continuously throughout the thickness of the plate, according to the volume fraction of the constituent materials based on the sigmoid FGM (S-FGM). The results show that critical temperature differences for the sigmoid functionally graded plates are generally lower than the corresponding values for homogeneous plates.

Bouazza et al [10] studied thermal buckling of functionally graded plates based on the using first order shear deformation theory. Material properties are varied continuously in the thickness direction according to a sigmoid distribution. The thermal buckling behaviours under uniform, linear and sinusoidal temperature rise across the thickness. The results show that critical temperature under sinusoidal temperature rise has the highest value in three cases, and that under linear temperature rise is higher than that under uniform temperature rise. Thermal buckling of functionally graded plates based on higher order theory was investigated by Javaheri and Eslami [11]. The study concludes that higher order shear deformation theory accurately predicts the behavior of functionally graded plates, whereas the classical plate theory overestimates buckling temperatures.

In this study, thermal buckling analyses of functionally graded materials subjected to lineaire temperature rise are examined by using first order shear deformation theory. Material properties are varied continuously in the thickness direction according to a power-law distribution. Numerical results are compared with results of the classic plate theory. Furthermore, the thermal buckling behaviors due to temperature field, volume fraction distributions, and system geometric parameters are studied, in detail.

2. Theoretical Formulation

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded properties. A FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function, exponential function, or sigmoid function to describe the volume fractions. In order to avoid the stress concentrations appear in one of the interfaces ,the power-law function is used in this study.

2.1. The material properties of P-FGM plates

In order to analyze P-FGM structures as shown in Fig. 1, the P-FGM function (Bao and Wang [12]) can be employed in this study. The volume fraction using power-law functions to ensure smooth distribution of stresses is defined.

$$V_f(z) = \left(z/h+1/2\right)^k \tag{1}$$

where is the thickness of the plate and is the material parameter that dictates the material variation profile through the thickness.

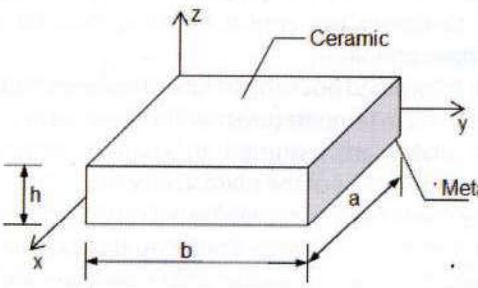

Fig. 1. Typical FGM square plate.

By using the rule of mixture, the material properties of the P-FGM can be calculated by

$$E(z) = V_f(z)E_c + [1 - V_f(z)]E \qquad (2)$$

where $E(z)$ denotes a generic material property such as modulus, E_c and E_m indicate the property of the top and bottom faces of the structure, respectively

Fig.2 shows that the variation of volume fraction in Eqs. (1) represents power-law distributions, and this FGM structure is thus called a P-FGM structure. Consider an elastic rectangular plate. The local coordinates and define the mid-plane of the plate, whereas the -axis originated at the middle surface of the plate is in the thickness direction. The material properties, Young's modulus, on the upper and lower surfaces are different but are pre-assigned according to the performance demands. However the Young's modulus of the plates and vary continuously only in the thickness direction (z -axis) i.e ., $E = E(z)$. It is called functionally graded material (FGM) plates. There have been numerous works on studying the response of FG plates made of isotropic elastic constituents with the homogenized material also modeled as isotropic elastic, the only other study on FG anisotropic plate (Pan [13]) has assumed that all elastic constants vary exponentially through the plate thickness at the same rate. It is highly unlikely that elastic modulus of a FG anisotropic plate will exhibit this property.

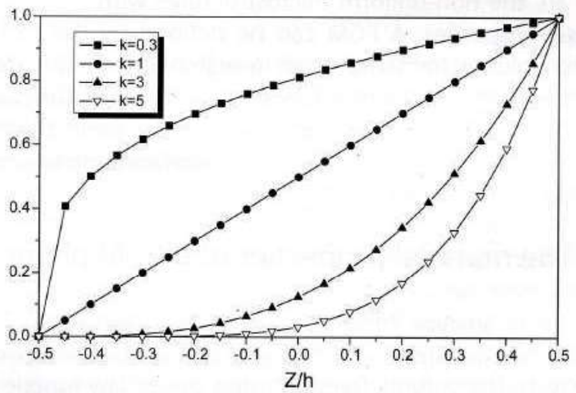

Fig. 2. The variation of volume fraction of P-FGM plate.

2.2 Stability equations

Assume that u, v, w denote the displacements of the neutral plane of the plate in x, y, z directions respectively; ϕ_x, ϕ_y denote the rotations of the normals to the plate midplane. According to the first order shear deformation theory, the strains of the plate can be expressed

$$\begin{aligned} &\varepsilon_x = u_{,x} + z\phi_{x,x} \qquad \varepsilon_y = \upsilon_{,y} + z\phi_{y,y} \\ &\gamma_{xy} = u_{,y} + \upsilon_{,x} + z(\phi_{x,y} + \phi_{y,x}) \\ &\gamma_{xz} = \phi_x + w_{,x} \qquad \gamma_{zy} = \phi_y + w_{,y} \end{aligned} \qquad (3)$$

The forces and moments per unit length of the plate expressed in terms of the stress components through the thickness are

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\, dz \;;\; M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z\, dz \;;\; Q_{ij} = \int_{-h/2}^{h/2} \tau_{ij}\, dz \qquad (4)$$

The nonlinear equations of equilibrium according to Von Karman's theory are given by:

$$\begin{aligned} &N_{x,x} + N_{xy,y} = 0 \\ &N_{y,y} + N_{xy,x} = 0 \\ &M_{x,x} + M_{xy,y} - Q_x = 0 \\ &M_{xy,x} + M_{y,y} - Q_y = 0 \\ &Q_{x,x} + Q_{y,y} + q + N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} = 0 \end{aligned} \qquad (5)$$

Using Eqs.(2), (3) and (4), and assuming that the temperature variation is linear , the equilibrium Eq. (5) may be reduced of one equations as

$$\nabla^4 w + \frac{2(1+v)}{E_1}\nabla^2(N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) \qquad (6)$$

$$-\frac{E_1(1-v^2)}{E_1E_3 - E_2^2}(N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) = 0$$

where

$$(E_1, E_2, E_3) = \int_{-h/2}^{h/2} (1, z, z^2)\, E\, dz \qquad (7)$$

$$(\Phi, \Theta) = \int_{-h/2}^{h/2} (1, z)\, E(z)\alpha(z)T(x,y,z)\, dz \qquad (8)$$

To establish the stability equations, the critical equilibrium method is used. Assuming that the state of stable equilibrium of a general plate under thermal load may be designated by . The displacement of the neighboring state is , where is an arbitrarily small increment of displacement. Substituting into Eq. (6) and subtracting the original equation, results in the following stability equation

$$\nabla^4 w_1 + \frac{2(1+\nu)}{E_1}\nabla^2 (N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) - \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}(N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) = 0 \quad (9)$$

where, N_x^0, N_y^0 and N_{xy}^0 refer to the pre-buckling force resultants

To determine the buckling temperature difference , the pre-buckling thermal forces should be found firstly. Solving the membrane form of equilibrium equations, gives the pre-buckling force resultants:

$$N_x^0 = -\frac{\Phi}{1-\nu}, \quad N_y^0 = -\frac{\Phi}{1-\nu}, \quad N_{xy}^0 = 0 \quad (10)$$

Substituting Eq(10) into Eq. (9), one obtains

$$\nabla^4 w_1 - \frac{2(1+\nu)}{E_1}\frac{\Phi}{1-\nu}\nabla^4 w_1 + \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}\frac{\Phi}{1-\nu}\nabla^2 w_1 = 0 \quad (11)$$

The simply supported boundary condition is defined as

$$\begin{aligned} &w_1 = 0, M_{x1} = 0, \phi_{y1} = 0 \text{ on } x = 0, a \\ &w_1 = 0, M_{y1} = 0, \phi_{x1} = 0 \text{ on } y = 0, b \end{aligned} \quad (12)$$

The following approximate solution is seen to satisfy both the governing equation and the boundary conditions

$$w_1 = c \sin(m\pi x/a)\sin(n\pi y/b) \quad (13)$$

where m, n are number of half waves in the x and y directions, respectively, and c is a constant coefficient. Substituting Eq. (13) into Eq. (11), and substituting for the thermal parameter from Eq. (8), yields

$$\Phi = \frac{(E_1E_3 - E_2^2)(1-\nu)\pi^2(m^2 + n^2B_a^2)E_1}{2(1+\nu)(E_1E_3 - E_2^2)\pi^2(m^2 + n^2B_a^2) + E_1^2a^2(1-\nu^2)} \quad (14)$$

where

$$B_a = a/b \quad (15)$$

2.3. Buckling Analysis

In this section, the thermal buckling behaviors of fully simply supported rectangular metal-ceramic plates under thermal environment are analyzed. The thermal load is assumed to be linear temperature rise through the thickness direction. The reference temperature is assumed to be 5°C. The effects of volume fraction index and geometric parameter a/h are investigated in each case. Typical values for alumina and aluminum are listed in Table 1 [9-11].

Table1 : Material properties [9-11].

Material	Property			
	E (GPa)	ν	$\alpha(1/°C)$	$k(W/mk)$
Aluminum	70	0.3	23e-6	204
Alumina	380	0.3	7.4e-6	10.4

2.3.1 Linear temperature rise

The temperature field under linear temperature rise through the thickness is assumed as

$$T(z) = \frac{\Delta T}{h}(z + h/2) + T_m \quad (16)$$

where z is the coordinate variable in the thickness direction which measured from the middle plane of the plate.

T_m is the metal temperature and ΔT is the temperature difference between ceramic surface and metal surface, i.e., . $\Delta T = T_c - T_m$ For this loading Φ case, the thermal parameter can be expressed as

$$\Phi = PT_m + X\Delta T \quad (17)$$

where

$$X = \int_{-h/2}^{h/2} E(z)\alpha(z)(z + h/2)dz \quad (18)$$

From Eq.(17) one has

$$\Delta T = \frac{\Phi - PT_m}{X} \quad (19)$$

The critical temperature difference is obtained for the values of that make the preceding expression a minimum. Apparently, when minimization methods are used, critical temperature difference is obtained for m=n=1.

Table2.Critical temperature for isotropic square plates subjected to different forms of temperature distribution

$$(a/h = 100, \alpha = 2\cdot 10^{-6}, \nu = 0.3)$$

Temperature distribution	Analytical [15]	FEM [14]	Present
Linear temperature rise	126.54	126	126.4739

3. Numerical Results and Discussion

First, Based on the derived formulation, a computer program is developed to study the behavior of FGM plates in thermal buckling to validation checks against the results available in the literature. The critical temperatures of simply supported, isotropic square plates subjected to linearly varying temperature distributions obtained using first order shear deformation theory are verified against the energy method based results of Gowda and Pandalai [15] and solution of Kri et al [14]based on finite element method using semiloof element, in Table 2. Both results are in excellent agreement.

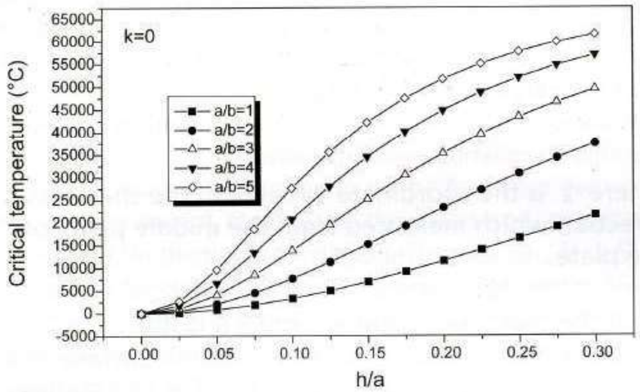

Fig. 3. Critical temperature gradient with respect to aspect ratio Ba and a/h under linear temperature rise

By solving Eq. (18), the critical temperature gradient can be obtained. Fig. 3 gives the variation of the critical temperature gradient of fully simply supported rectangular Aluminum–Alumina FGM plates under linear temperature rise. The responses are very similar comparing to those under uniform temperature rise (Bouazza et al [9,10]); however, the critical temperature gradient under linear temperature rise is higher than that under uniform temperature rise.

In this figure, the as aspect ratio a/b is increased, the critical temperature change increases. However, the critical temperature change decreases rapidly, when the geometric parameter a/h is decreased. In addition, the plates are thicker; the critical temperature change becomes higher.

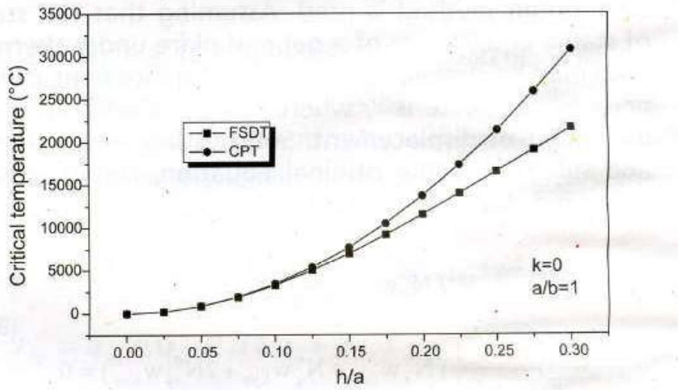

Fig.4. Comparison between temperature graphs vs. ratio h/a based on first order shear deformation theory, classic plate theory under linear temperature rise.`

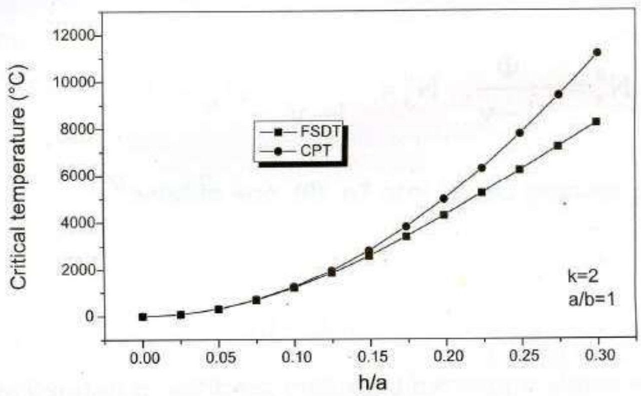

Fig.5. Comparison between temperature graphs vs. ratio h/a based on first order shear deformation theory, classic plate theory under linear temperature rise.

The critical temperature differences are calculated for functionally graded plates under linear temperature rise, and are plotted in Figs. 4–5. Figs. 4–5 shows the critical buckling temperature difference vs the thickness to span ratio h/a for different values of volume fraction exponent k(a/b=1). It is seen that the critical temperature difference increases monotonically as the relative thickness h/a increases. The values of the critical temperature differences calculated by using the first order shear deformation theory are lower than those calculated by using the classical plate theory, especially for thick plates.

CONCLUSIONS

The critical buckling temperatures of FGM plate have been obtained using a first order shear deformation theory. The results of the sample problem show good agreement with the literature values as seen from the validation checks. Based on the results reported here for various parameters of FGM plates, the following conclusions may be drawn.

(1) The critical buckling temperature differences for functionally graded plates are generally lower than the corresponding values for homogeneous plates. It is very important to check the strength of the functionally graded plate due to thermal buckling, although it has many advantages as a heat resistant material.

(2) The critical buckling temperature difference for a functionally graded plate is increased when the plate aspect ratio or the thickness to span ratio increases. However, it is decreased when the power law index k increases.

(3) Transverse shear deformation has considerable effect on the critical buckling temperature difference of functionally graded plate, especially for a thick plate or a plate with large aspect ratio.

(4) The critical temperature gradient under linear temperature rise is higher than that under uniform temperature rise.

REFERENCES

[1] Timoshenko SP, Gere JM. Theory of elastic stability. McGraw-Hill; 1961.

[2] Turvey GJ, Marshall IH. Buckling and postbuckling of composite plates. London: Chapman and Hall; 1995

[3] Leissa AW. Conditions for laminated plates to remain flat under inplane loading. Compos Struct 1986;6:261–70.

[4] Qatu MS, Leissa AW. Buckling or transverse deflections of unsymmetrically laminated plates subjected to in-plane loads. AIAA J 1993;31(1):189–94.

[5] Koizumi M. FGM activities in Japan. Compos, Part B 1997;28(1/2):1–4.

[6] Koizumi M, Niino M. Overview of FGM research in Japan. MRS Bull 1995;20(1):19–21.

[7] Kaysser WA, Ilschner B. FGM research activities in Europe. MRS Bull 1995;20(1):22–6.

[8] Birman V. Buckling of functionally graded hybrid composite plates. Proceeding of the 10th conference on engineering mechanics. vol. 2 1995 pp. 1199–202.

[9] Bouazza M., Tounsi A., Adda-Bedia E. A., Megueni A. Thermal buckling of sigmoid functionally graded plates using first order shear deformation theory, MAMERN09: 3rd International Conference on Approximation Methods and Numerical Modeling in Environment and Natural Resources Pau (France), June 8-11, 2009.

[10] Bouazza M., Tounsi A., Adda-Bedia E. A., Megueni A. Buckling Analysis of Functionally Graded Plates with Simply Supported Edges. Leonardo Journal of Sciences, ISSN 1583-0233, Issue 15, July-December 2009, p. 21-32.

[11] Javaheri R, Eslami MR. Thermal buckling of functionally graded plates based on higher orer theory. J Thermal Stresses 2002;25:603–25.

[12] Bao, G., Wang, L. Multiple cracking in functionally graded ceramic/metal coatings. International Journal of Solids and Structure 1995;32, 2853–2871.

[13] Pan E. Exact solution for functionally graded anisotropic composite laminates. J Compos Mater 2003;37:1903–20.

[14] Kari R. Thangaratnam, Palaninathan and j. Ramachandran,Thermal buckling of composite laminated plates. Computers & Structures vol.32, no. 5. pp. 1117-1124, 1989 Rimed in Great Britain.

[15] R. M. S. Gowda and K. A. V. Padalai, Thermal buckling of orthotropic plates. In Studies in Structural Mechanics (Edited by K. A. V. Padalai), pp. 9-44. IIT, Madras (1970).